1. An actuary is studying hurricane models. A year is classified as a high, medium, or low hurricane year with probabilities 0.1, 0.3, and 0.6, respectively. The numbers of hurricanes in high, medium, and low years follow Poisson distributions with means 20, 15, and 10, respectively. Calculate the variance of the number of hurricanes in a randomly selected year.
2. Points scored by a game participant can be modeled by *Z* = 3*X* + 2*Y* – 5. *X* and *Y* are independent random variables with Var (*X*) = 3 and Var (*Y*) = 4. Calculate Var (*Z*).
3. The independent random variables *X* and *Y* have the same mean. The coefficients of variation of *X* and *Y* are 3 and 4 respectively. Calculate the coefficient of variation of 
4. Two independent estimates are to be made on a building damaged by fire. Each estimate is normally distributed with mean 10*b* and variance  Calculate the probability that the first estimate is at least 20 percent higher than the second.
5. A delivery service owns two cars that consume 15 and 30 miles per gallon. Fuel costs 3 per gallon. On any given business day, each car travels a number of miles that is independent of the other and is normally distributed with mean 25 miles and standard deviation 3 miles. Calculate the probability that on any given business day, the total fuel cost to the delivery service will be less than 7.
6. A dental insurance company pays 100% of the cost of fillings and 70% of the cost of root canals. Fillings and root canals cost 50 and 500 each, respectively. The tables below show the probability distributions of the annual number of fillings and annual number of root canals for each of the company’s policyholders.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of Fillings | 0 | 1 | 2 | 3 |
| Probability | 0.60 | 0.20 | 0.15 | 0.05 |

|  |  |  |
| --- | --- | --- |
| Number of Roots Canal | 0 | 1 |
| Probability | 0.80 | 0.20 |

Calculate the expected annual payment per policyholder for fillings and root canals

1. Losses follow an exponential distribution with mean 1. Two independent losses are observed. Calculate the expected value of the smaller loss.
2. Annual windstorm losses, *X* and *Y*, in two different regions are independent, and each is uniformly distributed on the interval [0, 10]. Calculate the covariance of *X* and *Y*, given that *X* + *Y* < 10.
3. Random variables *X* and *Y* have joint distribution

|  |  |  |  |
| --- | --- | --- | --- |
|  | X=0 | X=1 | X=2 |
| Y=0 | 1/15 | a | 2/15 |
| Y=1 | a | b | a |
| Y=2 | 2/15 | a | 1/15 |

Let *a* be the value that minimizes the variance of *X.*  Calculate the variance of *Y*.

1. The intensity of a hurricane is a random variable that is uniformly distributed on the interval [0, 3]. The damage from a hurricane with a given intensity *y* is exponentially distributed with a mean equal to *y*. Calculate the variance of the damage from a random hurricane.
2. Let *X* be the annual number of hurricanes hitting Florida, and let *Y* be the annual number of hurricanes hitting Texas. *X* and *Y* are independent Poisson variables with respective means 1.70 and 2.30.Calculate 
3. On Main Street, a driver’s speed just before an accident is uniformly distributed on [5, 20]. Given the speed, the resulting loss from the accident is exponentially distributed with mean equal to three times the speed. Calculate the variance of a loss due to an accident on Main Street
4. The returns on two investments, *X* and *Y*, follow the joint probability density function



Calculate the maximum value of , 

1. Every day, the 30 employees at an auto plant each have probability 0.03 of having one accident and zero probability of having more than one accident. Given there was an accident, the probability of it being major is 0.01. All other accidents are minor. The numbers and severities of employee accidents are mutually independent. Let *X* and *Y* represent the numbers of major accidents and minor accidents, respectively, occurring in the plant today. Determine the joint moment generating function 
2. An individual experiences a loss due to property damage and a loss due to bodily injury. Losses are independent and uniformly distributed on the interval [0, 3]. Calculate the expected loss due to bodily injury, given that at least one of the losses is less than 1.
3. A couple takes out a medical insurance policy that reimburses them for days of work missed due to illness. Let *X* and *Y* denote the number of days missed during a given month by the wife and husband, respectively. The policy pays a monthly benefit of 50 times the maximum of *X* and *Y*, subject to a benefit limit of 100. *X* and *Y* are independent, each with a discrete uniform distribution on the set {0,1,2,3,4}. Calculate the expected monthly benefit for missed days of work that is paid to the couple.
4. A city with borders forming a square with sides of length 1 has its city hall located at the origin when a rectangular coordinate system is imposed on the city so that two sides of the square are on the positive axes. The density function of the population is



A resident of the city can travel to the city hall only along a route whose segments are parallel to the city borders. Calculate the expected value of the travel distance to the city hall of a randomly chosen resident of the city.

1. Let *X* denote the loss amount sustained by an insurance company’s policyholder in an auto collision. Let *Z* denote the portion of *X* that the insurance company will have to pay. An actuary determines that *X* and *Z* are independent with respective density and probability functions

 and 

Calculate the variance of the insurance company’s claim payment *ZX*.

1. The number of workplace accidents occurring in a factory on any given day is Poisson distributed with mean  . The parameter  is a random variable that is determined by the level of activity in the factory and is uniformly distributed on the interval [0,3]. Calculate the probability of one accident on a given day.
2. An investor invests 100 dollars in a stock. Each month, the investment has probability 0.5 of increasing by 1.10 dollars and probability 0.5 of decreasing by 0.90 dollars. The changes in price in different months are mutually independent. Calculate the probability that the investment has a value greater than 91 dollars at the end of month 100.
3. A company provides a death benefit of 50,000 for each of its 1000 employees. There is a 1.4% chance that any one employee will die next year, independent of all other employees. The company establishes a fund such that the probability is at least 0.99 that the fund will cover next year’s death benefits. Calculate, using the Central Limit Theorem, the smallest amount of money, rounded to the nearest 50 thousand, that the company must put into the fund.
4. The number of minor surgeries, *X*, and the number of major surgeries, *Y*, for a policyholder, this decade, has joint cumulative distribution function for nonnegative integers *x* and *y*. Calculate the probability that the policyholder experiences exactly three minor surgeries and exactly three major surgeries this decade.
5. Skateboarders A and B practice one difficult stunt until becoming injured while attempting the stunt. On each attempt, the probability of becoming injured is *p*, independent of the outcomes of all previous attempts. Let *F*(*x*, *y*) represent the probability that skateboarders A and B make no more than *x* and *y* attempts, respectively, where *x* and *y* are positive integers. It is given that *F*(2, 2) = 0.0441. Calculate *F*(1, 5).
6. An insurance company sells automobile liability and collision insurance. Let *X* denote the percentage of liability policies that will be renewed at the end of their terms and *Y* the percentage of collision policies that will be renewed at the end of their terms. *X* and *Y* have a joint cumulative distribution



Calculate the probability that at least 80% of liability policies and 80% of collision policies will be renewed at the end of their terms.

1. A car and a bus arrive at a railroad crossing at times independently and uniformly distributed between 7:15 and 7:30. A train arrives at the crossing at 7:20 and halts traffic at the crossing for five minutes. Calculate the probability that the waiting time of the car or the bus at the crossing exceeds three minutes.
2. Batteries A and B have lifetimes that are independent and exponentially distributed with a common mean of *m* years. The probability that battery B outlasts battery A by more than one year is 0.33. Calculate *m*.
3. Let *X* be a continuous random variable with probability density function



where  .Let *Y* be the smallest integer greater than or equal to *X*. Determine the probability function of *Y*.

1. The number of boating accidents a policyholder experiences this year is modeled by a Poisson random variable with variance 0.10. An insurer reimburses only the first accident. Let *Y* be the number of unreimbursed accidents the policyholder experiences this year and let *p* be the probability function of *Y*. Determine *p*(*y*).
2. A home owner’s insurance policy covers losses due to theft, with a deductible of 3. Theft losses are uniformly distributed on [0, 10]. Determine the moment generating function, M(*t*), for *t* ≠ 0, of the claim payment on a theft.
3. Ten cards from a deck of playing cards are in a box: two diamonds, three spades, and five hearts. Two cards are randomly selected without replacement. Calculate the variance of the number of diamonds selected, given that no spade is selected.
4. The number of claims *X* on a health insurance policy is a random variable with  and  . Calculate the standard deviation of the number of claims.
5. A company’s annual profit, in billions, has a normal distribution with variance equal to the cube of its mean. The probability of an annual loss is 5%. Calculate the company’s expected annual profit.
6. Under a liability insurance policy, losses are uniformly distributed on [0, *b*], where *b* is a positive constant. There is a deductible of *b*/2. Calculate the ratio of the variance of the claim payment (greater than or equal to zero) from a given loss to the variance of the loss.
7. A government employee’s yearly dental expense follows a uniform distribution on the interval from 200 to 1200. The government’s primary dental plan reimburses an employee for up to 400 of dental expense incurred in a year, while a supplemental plan pays up to 500 of any remaining dental expense. Let *Y* represent the yearly benefit paid by the supplemental plan to a government employee. Calculate Var(*Y*).
8. The number of tornadoes in a given year follows a Poisson distribution with mean 3. Calculate the variance of the number of tornadoes in a year given that at least one tornado occurs.
9. For a certain insurance company, 10% of its policies are Type A, 50% are Type B, and 40% are Type C. The annual number of claims for an individual Type A, Type B, and Type C policy follow Poisson distributions with respective means 1, 2, and 10. Let *X* represent the annual number of claims of a randomly selected policy. Calculate the variance of *X*.
10. Losses, *X*, under an insurance policy are exponentially distributed with mean 10. For each loss, the claim payment *Y* is equal to the amount of the loss in excess of a deductible d> 0. Calculate Var(*Y*).
11. The loss *L* due to a boat accident is exponentially distributed. Boat insurance policy A covers up to 1 unit for each loss. Boat insurance policy B covers up to 2 units for each loss. The probability that a loss is fully covered under policy B is 1.9 times the probability that it is fully covered under policy A. Calculate the variance of *L*
12. A large university will begin a 13-day period during which students may register for that semester’s courses. Of those 13 days, the number of elapsed days before a randomly selected student registers has a continuous distribution with density function *f*(*t*) that is symmetric about *t* = 6.5 and proportional to 1/(*t* + 1) between days 0 and 6.5. A student registers at the 60th percentile of this distribution. Calculate the number of elapsed days in the registration period for this student
13. A gun shop sells gunpowder. Monthly demand for gunpowder is normally distributed, averages 20 pounds, and has a standard deviation of 2 pounds. The shop manager wishes to stock gunpowder inventory at the beginning of each month so that there is only a 2% chance that the shop will run out of gunpowder (i.e., that demand will exceed inventory) in any given month. Calculate the amount of gunpowder to stock in inventory, in pounds.
14. Losses incurred by a policyholder follow a normal distribution with mean 20,000 and standard deviation 4,500. The policy covers losses, subject to a deductible of 15,000. Calculate the 95th percentile of losses that exceed the deductible.
15. An insurance policy will reimburse only one claim per year. For a random policyholder, there is a 20% probability of no loss in the next year, in which case the claim amount is 0. If a loss occurs in the next year, the claim amount is normally distributed with mean 1000 and standard deviation 400. Calculate the median claim amount in the next year for a random policyholder.
16. A motorist just had an accident. The accident is minor with probability 0.75 and is otherwise major. Let *b* be a positive constant. If the accident is minor, then the loss amount follows a uniform distribution on the interval [0, *b*]. If the accident is major, then the loss amount follows a uniform distribution on the interval [*b*, 3*b*]. The median loss amount due to this accident is 672. Calculate the mean loss amount due to this accident.
17. For a certain health insurance policy, losses are uniformly distributed on the interval [0, 450]. The policy has a deductible of *d* and the expected value of the unreimbursed portion of a loss is 56. Calculate *d*.
18. The number of burglaries occurring on Burlington Street during a one-year period is Poisson distributed with mean 1. Calculate the expected number of burglaries on Burlington Street in a one-year period, given that there are at least two burglaries.
19. A company provides each of its employees with a death benefit of 100. The company purchases insurance that pays the cost of total death benefits in excess of 400 per year. The number of employees who will die during the year is a Poisson random variable with mean 2. Calculate the expected annual cost to the company of providing the death benefits, excluding the cost of the insurance.
20. A company has purchased a policy that will compensate for the loss of revenue due to severe weather events. The policy pays 1000 for each severe weather event in a year after the first two such events in that year. The number of severe weather events per year has a Poisson distribution with mean 1. Calculate the expected amount paid to this company in one year.
21. Losses under an insurance policy are exponentially distributed with mean 4. The deductible is 1 for each loss. Calculate the median amount that the insurer pays a policyholder for a loss under the policy.
22. A certain town experiences an average of 5 tornadoes in any four-year period. The number of years from now until the town experiences its next tornado as well as the number of years between tornados have identical exponential distributions and all such times are mutually independent Calculate the median number of years from now until the town experiences its next tornado.
23. A factory tests 100 light bulbs for defects. The probability that a bulb is defective is 0.02. The occurrences of defects among the light bulbs are mutually independent events. Calculate the probability that exactly two are defective given that the number of defective bulbs is two or fewer.
24. Patients in a study are tested for sleep apnea, one at a time, until a patient is found to have this disease. Each patient independently has the same probability of having sleep apnea. Let *r* represent the probability that at least four patients are tested. Determine the probability that at least twelve patients are tested given that at least four patients are tested.
25. A flood insurance company determines that *N*, the number of claims received in a month, is a random variable with ,for n=0,1,2,… The numbers of claims received in different months are mutually independent. Calculate the probability that more than three claims will be received during a consecutive two-month period, given that fewer than two claims were received in the first of the two months.
26. Events E and F are independent. P[E] = 0.84 and P[F] = 0.65. Calculate the probability that exactly one of the two events occurs.
27. Let *X* be a random variable with density function



Calculate 

1. The lifetime of a machine part is exponentially distributed with a mean of five years. Calculate the mean lifetime of the part, given that it survives less than ten years.
2. Individuals purchase both collision and liability insurance on their automobiles. The value of the insured’s automobile is *V*. Assume the loss *L* on an automobile claim is a random variable with cumulative distribution function



Calculate the probability that the loss on a randomly selected claim is greater than the value of the automobile

1. The annual profit of a life insurance company is normally distributed. The probability that the annual profit does not exceed 2000 is 0.7642. The probability that the annual profit does not exceed 3000 is 0.9066. Calculate the probability that the annual profit does not exceed 1000.
2. The time until the next car accident for a particular driver is exponentially distributed with a mean of 200 days. Calculate the probability that the driver has no accidents in the next 365 days, but then has at least one accident in the 365-day period that follows this initial 365-day period.
3. Losses due to burglary are exponentially distributed with mean 100. The probability that a loss is between 40 and 50 equals the probability that a loss is between 60 and *r*, with *r* > 60. Calculate *r*.
4. Four distinct integers are chosen randomly and without replacement from the first twelve positive integers. *X* is the random variable representing the second smallest of the four selected integers, and *p* is the probability function of *X*. Determine *p*(*x*) for *x* = 2,3,…,10.
5. A representative of a market research firm contacts consumers by phone in order to conduct surveys. The specific consumer contacted by each phone call is randomly determined. The probability that a phone call produces a completed survey is 0.25. Calculate the probability that more than three phone calls are required to produce one completed survey.
6. A group of 100 patients is tested, one patient at a time, for three risk factors for a certain disease until either all patients have been tested or a patient tests positive for more than one of these three risk factors. For each risk factor, a patient tests positive with probability *p*, where 0 < *p* < 1. The outcomes of the tests across all patients and all risk factors are mutually independent. Determine an expression for the probability that exactly *n* patients are tested, where *n* is a positive integer less than 100.
7. The number of traffic accidents occurring on any given day in Coralville is Poisson distributed with mean 5. The probability that any such accident involves an uninsured driver is 0.25, independent of all other such accidents. Calculate the probability that on a given day in Coralville there are no traffic accidents that involve an uninsured driver.
8. An insurance company studies back injury claims from a manufacturing company. The insurance company finds that 40% of workers do no lifting on the job, 50% do moderate lifting and 10% do heavy lifting. During a given year, the probability of filing a claim is 0.05 for a worker who does no lifting, 0.08 for a worker who does moderate lifting and 0.20 for a worker who does heavy lifting. A worker is chosen randomly from among those who have filed a back injury claim. Calculate the probability that the worker’s job involves moderate or heavy lifting.
9. At a mortgage company, 60% of calls are answered by an attendant. The remaining 40% of callers leave their phone numbers. Of these 40%, 75% receive a return phone call the same day. The remaining 25% receive a return call the next day. Of those who initially spoke to an attendant, 80% will apply for a mortgage. Of those who received a return call the same day, 60% will apply. Of those who received a return call the next day, 40% will apply. Calculate the probability that a person initially spoke to an attendant, given that he or she applied for a mortgage.
10. A drawer contains four pairs of socks, with each pair a different color. One sock at a time is randomly drawn from the drawer until a matching pair is obtained. Calculate the probability that the maximum number of draws is required.
11. Six claims are to be randomly selected from a group of thirteen different claims, which includes two worker’s compensation claims, four homeowner’s claims and seven auto claims. Calculate the probability that the six claims selected will include one worker’s compensation claim, two homeowner’s claims and three auto claims.
12. A machine has two parts labelled A and B. The probability that part A works for one year is 0.8 and the probability that part B works for one year is 0.6. The probability that at least one-part works for one year is 0.9. Calculate the probability that part B works for one year, given that part A works for one year.
13. A health insurance policy covers visits to a doctor’s office. Each visit costs 100. The annual deductible on the policy is 350. For a policy, the number of visits per year has the following probability distribution:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of Visits | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Probability | 0.60 | 0.15 | 0.10 | 0.08 | 0.04 | 0.02 | 0.01 |

A policy is selected at random from those where costs exceed the deductible. Calculate the probability that this policyholder had exactly five office visits.

1. Four letters to different insureds are prepared along with accompanying envelopes. The letters are put into the envelopes randomly. Calculate the probability that at least one letter ends up in its accompanying envelope.
2. The annual numbers of thefts a homeowner’s insurance policyholder experiences are analyzed over three years. Define the following events:

i) A = the event that the policyholder experiences no thefts in the three years.

ii) B = the event that the policyholder experiences at least one theft in the second year.

iii) C = the event that the policyholder experiences exactly one theft in the first year.

iv) D = the event that the policyholder experiences no thefts in the third year.

v) E = the event that the policyholder experiences no thefts in the second year, and at least one theft in the third year.

Determine which three events satisfy the condition that the probability of their union equals the sum of their probabilities. Let A, B, and C be events such that *P*[A] = 0.2, *P*[B] = 0.1, and *P*[C] = 0.3. The events A and B are independent, the events B and C are independent, and the events A and C are mutually exclusive. Calculate 

A life insurance company has found there is a 3% probability that a randomly selected application contains an error. Assume applications are mutually independent in this respect. An auditor randomly selects 100 applications. Calculate the probability that 95% or less of the selected applications are error-free.

A state is starting a lottery game. To enter this lottery, a player uses a machine that randomly selects six distinct numbers from among the first 30 positive integers. The lottery randomly selects six distinct numbers from the same 30 positive integers. A winning entry must match the same set of six numbers that the lottery selected. The entry fee is 1, each winning entry receives a prize amount of 500,000, and all other entries receive no prize. Calculate the probability that the state will lose money, given that 800,000 entries are purchased

A company sells two types of life insurance policies (P and Q) and one type of health insurance policy. A survey of potential customers revealed the following:

i) No survey participant wanted to purchase both life policies.

ii) Twice as many survey participants wanted to purchase life policy P as life policy Q.

iii) 45% of survey participants wanted to purchase the health policy.

iv) 18% of survey participants wanted to purchase only the health policy.

v) The event that a survey participant wanted to purchase the health policy was independent of the event that a survey participant wanted to purchase a life policy. Calculate the probability that a randomly selected survey participant wanted to purchase exactly one policy.

An actuary has done an analysis of all policies that cover two cars. 70% of the policies are of type A for both cars, and 30% of the policies are of type B for both cars. The number of claims on different cars across all policies are mutually independent. The distributions of the number of claims on a car are given in the following table.

|  |  |  |
| --- | --- | --- |
| Number of Claims | Type A | Type B |
| 0 | 40% | 25% |
| 1 | 30% | 25% |
| 2 | 20% | 25% |
| 3 | 10% | 25% |

Four policies are selected at random. Calculate the probability that exactly one of the four policies has the same number of claims on both covered cars

Bowl I contains eight red balls and six blue balls. Bowl II is empty. Four balls are selected at random, without replacement, and transferred from bowl I to bowl II. One ball is then selected at random from bowl II. Calculate the conditional probability that two red balls and two blue balls were transferred from bowl I to bowl II, given that the ball selected from bowl II is blue.

Each week, a subcommittee of four individuals is formed from among the members of a committee comprising seven individuals. Two subcommittee members are then assigned to lead the subcommittee, one as chair and the other as secretary. Calculate the maximum number of consecutive weeks that can elapse without having the subcommittee contain four individuals who have previously served together with the same subcommittee chair.

An actuary compiles the following information from a portfolio of 1000 homeowner’s insurance policies:

i) 130 policies insure three-bedroom homes.

ii) 280 policies insure one-story homes.

iii) 150 policies insure two-bath homes.

iv) 30 policies insure three-bedroom, two-bath homes.

v) 50 policies insure one-story, two-bath homes.

vi) 40 policies insure three-bedroom, one-story homes.

vii) 10 policies insure three-bedroom, one-story, two-bath homes.

Calculate the number of homeowner’s policies in the portfolio that insure neither one-story nor two-bath nor three-bedroom homes.

An insurance policy pays 100 per day for up to three days of hospitalization and 50 per day for each day of hospitalization thereafter*.* The number of days of hospitalization, *X*, is a discrete random variable with probability function



Calculate the expected payment for hospitalization under this policy.

A profile of the investments owned by an agent’s clients follows:

i) 228 own annuities.

ii) 220 own mutual funds.

iii) 98 own life insurance and mutual funds.

iv) 93 own annuities and mutual funds.

v) 16 own annuities, mutual funds, and life insurance.

vi) 45 more clients own only life insurance than own only annuities.

vii) 290 own only one type of investment (i.e., annuity, mutual fund, or life insurance).

Calculate the agent’s total number of clients.

An insurance agent’s files reveal the following facts about his policyholders:

i) 243 own auto insurance.

ii) 207 own homeowner insurance.

iii) 55 own life insurance and homeowner insurance.

iv) 96 own auto insurance and homeowner insurance.

v) 32 own life insurance, auto insurance and homeowner insurance.

vi) 76 more clients own only auto insurance than only life insurance.

vii) 270 own only one of these three insurance products.

Calculate the total number of the agent’s policyholders who own at least one of these three insurance products.